APPENDIX E: DC/IP/MT THEORY

INTRODUCTION

Resistivity is among the most variable of all geophysical parameters, with a dynamic range exceeding 10^6. Because most minerals are fundamentally insulators, with the exception of massive accumulations of metallic and submetallic ores (electronic conductors) which are rare occurrences, the resistivity of rocks depends primarily on their porosity, permeability and particularly the salinity of fluids contained (ionic conduction), according to Archie's Law. In contrast, the chargeability responds to the presence of polarizeable minerals (metals, submetallic sulphides and oxides, and graphite), in concentrations as small as parts per hundred. Both the concentration of individual chargeable grains present and their distribution within subsurface current flow paths are significant in controlling the level of response. The relationship of chargeability to metallic content is straightforward, while the influence of mineral distribution can be understood in geologic terms by considering two similar, hypothetical volumes of rock in which fractures constitute the primary current flow paths. In one, sulphides occur predominantly along fracture surfaces. In the second, the same volume percent of sulphides are disseminated throughout the rock. The second example will, in general, have significantly lower intrinsic chargeability.

More detailed descriptions on the theory and application of the IP/Resistivity method can be found in Van Blaricom (1992) and Telford et al. (1976).

HALVERSON-WAIT CHARGEABILITY

The Titan-24 DCIP chargeability decays are described using the Halverson-Wait spectral model (Halverson et al., 1981), which is similar to the Cole-Cole model proposed by Pelton et al. (1978). The Cole-Cole model is a simple relaxation model that fits complex (frequency-dependant) resistivity results.

The time domain chargeability, originally proposed by Siegel (1959), is defined (Telford et al., 1976) as

$$M = \frac{1}{V_C} \int_{t1}^{t2} V(t)\,dt$$

Where $V(t)$ is the residual or secondary voltage at a time t, that is decaying after the current is cut off, between time t_1 and t_2, with the steady voltage V_C during the current flow interval. The ratio $V(t)/V_C$ is expressed in millivolts per volt.

In the frequency domain, the "frequency effect" is defined as:

$$fe = (\rho_{DC} - \rho_{AC}) / \rho_{AC}$$

Where ρ_{DC} and ρ_{AC} are apparent resistivities measured at d.c. and "very high" frequency, usually in the 0.1 to 10 Hz range.

The Cole-Cole model for the chargeability m, as defined by Pelton et al. (1978) is given by the following:

$$Z(\omega) = R_0\left[1 - m\left(1 - \frac{1}{1+(i\omega\tau)^C}\right)\right]$$

Where $Z(\omega)$ is the complex impedance, R_0 is the DC resistivity, m is the chargeability in volts per volt, ω

is the angular frequency in Hz, τ is the time constant in seconds, and ***c*** is the frequency dependence (unitless). The latter two physical properties describe the shape of the decay curve in time domain or the phase spectrum in frequency domain, and commonly range between 0.01s to +100s and 0.1 to +0.5, respectively (Johnson, 1984).

The Halverson-Wait model was proposed by Halverson et al. (1981) as an extension to the Wait (1959) model of the impedance of "volume loading" of spheres, given by:

$$Z(\omega)=\frac{\rho}{G}\left[1-3v\left(1-\frac{3\delta}{1+2\delta}\right)\right]$$

Where ***G*** is a geometric factor, ***ρ*** is the resistivity of the media, ***v*** is the volume loading (the volume fraction of chargeable "spheres"), δ is the sphere surface impedance. The Wait model was designed to provide an explanation of the differences in the shape of decay curves from different polarizeable targets, but does not describe very well the physical attributes of the rocks.

The Halverson-Wait model expands the Wait coated sphere IP model to include a new formulation of the sulphide-rock interface impedance, based on field studies and laboratory tests on samples. It is closely correlated to the Pelton et al. (1978) Cole-Cole model and is given by:

$$Z(\omega)=\frac{\rho}{G}\left[1-3v\left(1-\frac{3/2}{1+r[i\omega]^{K}}\right)\right]$$

Where ***r*** is the sphere radius and is equivalent to τ - the Cole-Cole time constant (*r* = τ^{K}). The ***v*** volume loading compares well to ***m*** – the Cole-Cole chargeability (see equation below) – and the exponent ***k*** is equal to ***c*** - the Cole-Cole frequency dependence (Halverson et al., 1983). For sulphide systems, the *r*-factor reflects the size or interconnection of the sulphide grains and the *k*-factor reflects the electrical characteristics of the sulphide surfaces. An example of time domain Halverson-Wait model responses is shown in Figure G.1.

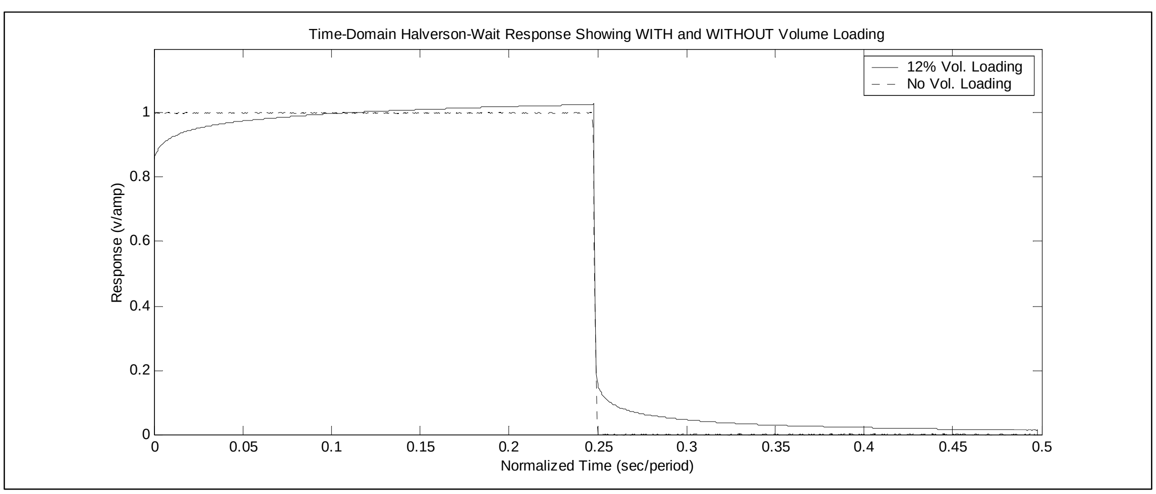

Figure G.1: Polarizeable versus Non-Polarizeable TDIP Response using Halverson-Wait Model

In practice the Titan chargeability decays are fit to a Halverson-Wait model. In order to solve for the volume loading *v*, the *r*-factor and *k*-factor are set to the standard (typical) Halverson-Wait values of 1.0 and 0.2, respectively. In the Halverson-Wait model the theoretical PFE (for infinite bandwidth), which equates to the theoretical chargeability in the Cole-Cole equation, is thereby defined by the volume loading:

$$PFE_0 = m_0 = \frac{9v}{(2+3v)}$$

and ***m*** is output in units of milliradians.

INVERSION THEORY

An excellent overview and introduction to both the philosophy and use of inversions in geophysics is available on the University of British Columbia (UBC) website (http://www.eos.ubc.ca/ubcgif/; Oldenburg et al., 1998).

Several points, detailed on the website, are crucial to understanding the Titan-24 approach to exploration:

- Inversion is a powerful 'tool', not a 'solution'.
- Inversion is not normally "unique". Given noisy and incomplete data of inherently limited resolution there are usually an 'infinite' range of models that 'fit' the data equally well. Recognition of this inherent non-uniqueness is why inversion must be viewed as a tool rather than a solution. Understanding and exploration of this non-uniqueness is an important part of the interpretive process.
- Inversion finds a model that 'fits' the data. The precise definition of 'fit' can be critical in the actual model that is found.
- The inversion depends on the data, and the data errors. The importance of the data errors is often overlooked.
- Inversion depends on a "model norm" – the mathematical definition of which model the inversion should try to find. This definition is almost as important as the actual data in determining the final inversion model.

Mathematically, inversion is the process of minimizing a function. The choice of which function to minimize ultimately defines the inversion model. Schematically, this function might be expressed:

$$\phi = \phi_d + \beta\,\phi_m = \text{(misfit)} + \beta\,\text{(model norm)}$$

$$0 < \beta < \infty \quad \text{is a constant}$$

This defines a function to be minimized that consists of some function that minimizes the data misfit, combined with some function that finds a "smooth" model. Beta represents a relative weighting between fitting the data and smoothing the model.

Clearly, the data misfit function must be defined in more detail. One approach might be:

$$\phi_d = \sum_{i=1}^{N} \left(\frac{F_i[m] - d_i^{obs}}{\varepsilon_i} \right)^2$$

This function defines the data misfit as the sum of the individual misfits squared, normalized by the errors associated with each data point. It is a very common, and stable, definition of the data misfit.

Further to the UBC website information, the errors depend on many factors. The most common measure of data errors is simply the repeatability of the voltage and current measurements in the field. This may be misleading as there are also “errors” associated with electrode positioning, geologic complexity (2D vs 3D, coupling of shallow and deeper structure), and numerical computation errors in the calculation of model responses.

Another point is the importance of not overestimating the data errors and subsequently under-fitting the data. Most geophysical techniques, and particularly electrical techniques, have large responses to shallow structure. This is expressed as “pant legs” in DC/IP, or “statics” in MT. The responses to deep structure is generally a very subtle component of the data, compared to the dominate responses from shallow structure. High quality data and appropriate treatment of the data are then necessary to extract the subtle deeper information.

The model misfit function must also be defined in more detail. One of the most flexible definitions is the one used by UBC:

$$\phi_m(m,m_0)=\alpha_s\int_{vol}(m-m_0)^2dv+\alpha_x\int_{vol}\left(\frac{\partial(m-m_0)}{\partial x}\right)^2dv+\alpha_z\int_{vol}\left(\frac{\partial(m-m_0)}{\partial z}\right)^2dv$$

In this definition there are three components to the “model norm” (or “smoothness” constraint, or “regularization”), each of which contains an α constant (α_S, α_X, α_Z) that are commonly referred to as “alpha parameters”. The first component is simply an overall difference between the model and a “target” model, the second component is a horizontal smoothness, and the third component is a vertical smoothness. The three “alpha” parameters (α_S, α_X, α_Z) represent a relative weighting of each component. A fourth variable, m_0, refers to the starting or reference model – either a half-space or geophysical constraint – that also has a profound influence on the model-misfit.

The UBC website provides an excellent example of the importance of selecting an appropriate “model norm”, reproduced in Figure I.2

In this example the expected response of the top figure was computed. These ‘data’ were then inverted six times, using different “model norms” (α_S, α_X, $\alpha_{Z,}$ m_0). The lower six figures show the range of valid inversion models that can be produced. Note that six of these models are essentially mathematically equivalent, they all “fit” the data.

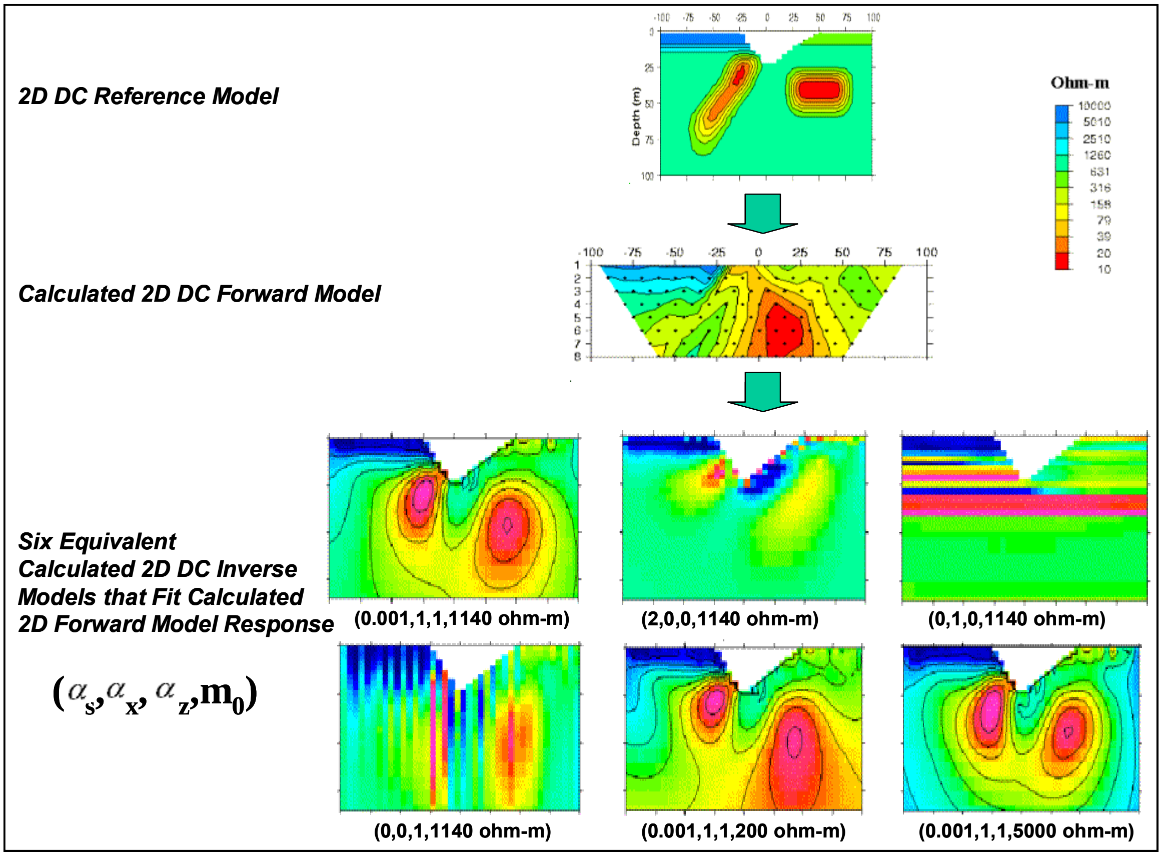

Figure I.2: Effects of Model Norm and Starting Model on Inversion Results (modified after Oldenburg, et al.,1998).

An important philosophy, driving much of the academic communities approach to inversion for the last two decades, is that the "best" model is the "smoothest" model consistent with the data. There are good reasons for taking this approach. However, from an exploration viewpoint this philosophy can be rephrased to "find the model with the least exploration value" – perhaps not reflecting the real goal of an exploration program.

Recently, several groups have taken major steps towards developing inversion approaches more tuned to exploration needs. Instead of using "smooth" model norms, they are being replaced with "focused (minimum transition zone) inversion, or smoothing to a geologic "target" model.

For exploration smoothing to a geologic target model makes sense. It requires good geologic control, and some understanding of the rock physical properties. There are three drawbacks to the geologic target approach:

- The geologic information is incomplete or inaccurate.
- Physical property data are incomplete.
- It is difficult to determine whether the geophysical data support the geologic model, or simply provide no information.

The most sensible approach is to combine smooth model inversion with geologic target inversion. For now, we are focusing on providing inversions using both approaches. It is up to the project geologist and geophysicist to review these inversions and develop a final interpretation.

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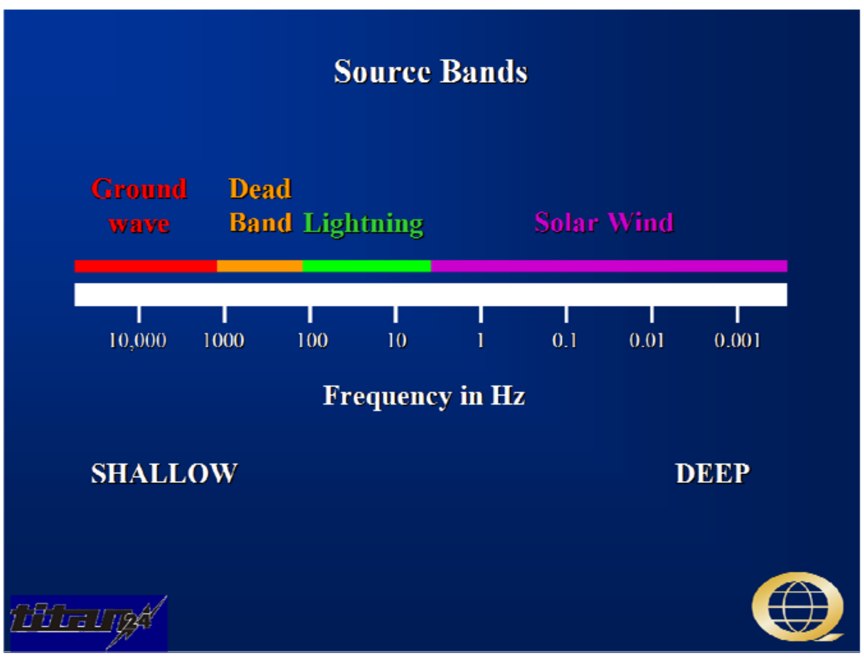
INTRODUCTION TO THE MAGNETOTELLURIC (MT) METHOD

The magnetotelluric (MT) method utilizes time-variations in the Earth's natural electric (E) and magnetic (H) fields to image the resistivity of the subsurface structure. The natural electromagnetic (EM) signals are assumed to be of plane-wave source over the frequency range with which the MT surveys are usually carried out. The plane-wave source is much simpler to model than complex transmitter geometries and signals.

The E and H fields are measured over a broad range of frequencies. Typically, the frequencies can range from above 10 kHz to below 0.001Hz. High frequency signals are attenuated more rapidly in the subsurface. Therefore, high frequency data are indicative of shallow resistivity structure while low frequency data are indicative of deep resistivity structure.

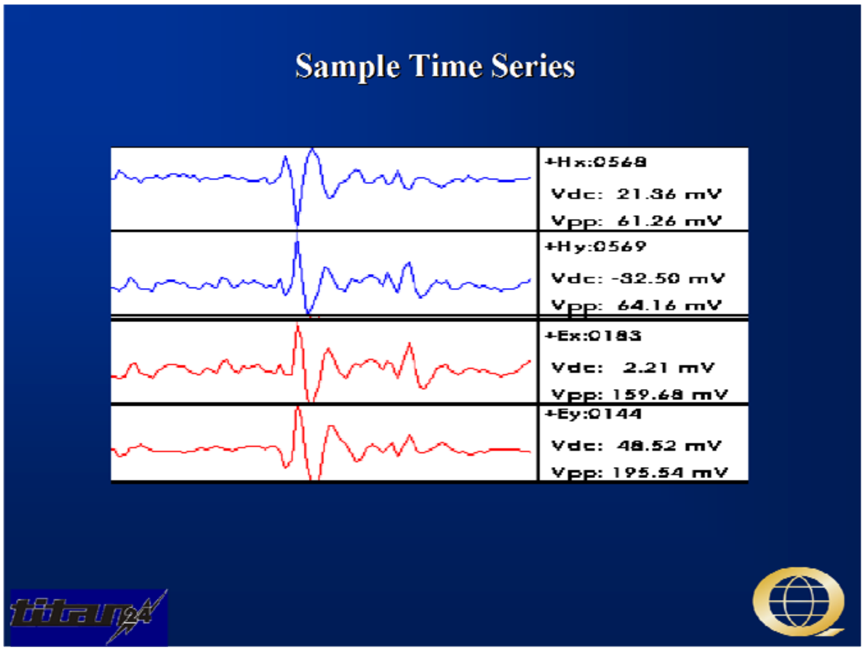
At frequencies below 1Hz the EM signal source is due to oscillations of the Earth's ionosphere as it interacts with the solar wind. At frequencies above 1Hz the signal source is due to worldwide lightning activities. There is a lack of natural signal around 1Hz, often referred to as the "hole". Modern 24-bit recording hardware and signal processing techniques have largely eliminated the data quality degradations that have been traditionally seen around the 1Hz signal hole.

Between about 8Hz and 300Hz the signal from worldwide lightning activity propagates in a "resonant" cavity (the resistive atmosphere) between the conductive ionosphere and the Earth's surface. Above 3 kHz the signal propagates as a ground wave. Between 300Hz and 3 kHz there is a "dead-band" where the signal does not propagate well. Despite hardware and signal processing improvements this dead-band remains problematic. When signal (atmospheric activity) is present within several hundreds of miles of the survey area the data quality improves. When no signal is being generated in the vicinity data quality is poor.

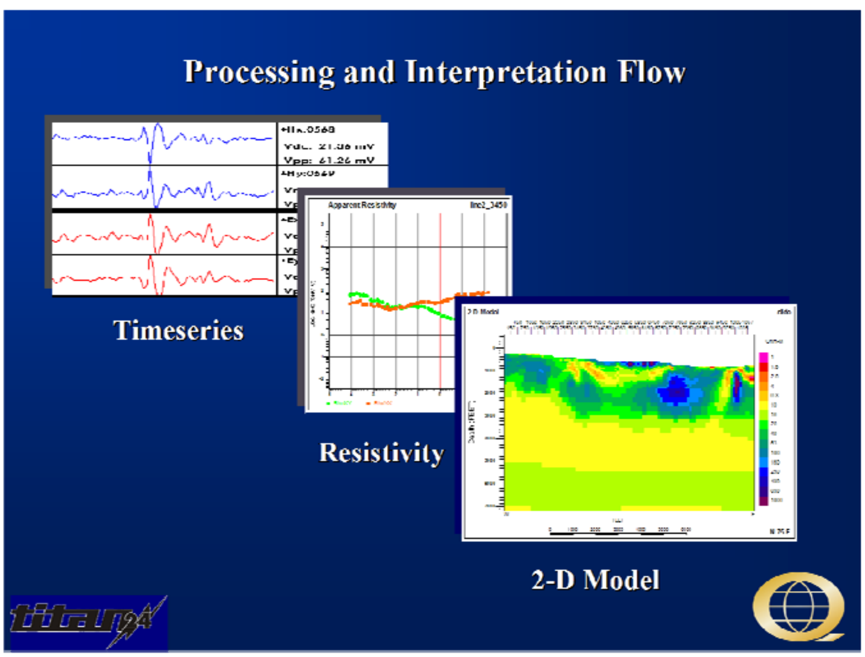


Both the electric and magnetic fields are measured. The measured field strengths depend on the ionosphere and lightning, and are essentially of random nature. While the E and H fields are random the ratio of the fields depends on the subsurface resistivity structure. The magnetic field is perpendicular to the electric field for a homogeneous and 1D earth resistivity structures. However, it is possible for complex subsurface resistivity structure to rotate the fields, and full tensor data are usually measured.

In the field surveys, the electric and magnetic fields are measured as a function of time. The electric field is measured using two orthogonal grounded dipoles. The magnetic field is also measured using induction coils parallel to the electric dipoles.



Extracting the subsurface resistivity structure from the measured magnetic and electric fields is a multi-step process. First time series processing techniques are used to derive impedance tensor from the electric and magnetic fields. The impedance tensor is then transformed into apparent resistivity and phase data. In interpretation stage, first inversion techniques are used to invert the apparent resistivity and phase data to a subsurface resistivity image. Finally, the resistivity image must be interpreted in terms of geologic units.



In time series processing, the measured magnetic and electric fields are Fourier transformed into the frequency domain. Calibration curves are applied to the measured fields to remove the measurement system response. The Fourier coefficients represent the amplitude and phase of the electric and magnetic fields as a function of frequency.

A variety of complex signal processing techniques are used to minimize noise and bias in the estimation of geophysical parameters from the measured fields. The approaches include:

1. Spatial isolation of noise. A remote reference magnetic station is used to separate signal from local noise in the magnetic field data.
2. Coherency sieves to find coherent signal. First the local and remote magnetic field measurements are compared and coherent signal kept. Then the local magnetic and electric fields are compared for coherency.
3. Frequency isolation of noise. Long Fourier transforms are used to provide extremely sharp isolation of noise in frequency.
4. Time isolation of noise. Short Fourier transforms are used to remove noise that is isolated in time (noise spikes, or noise that is randomly turning off and on).
5. Robust statistics that minimize biasing effects of a few isolated measurements.

Once the time series processing is complete geophysical parameters can be estimated. The primary geophysical parameters for MT are typically the apparent resistivity versus frequency and phase versus frequency.

The depth of penetration of the EM signal depends on the frequency of the data and the resistivity of the subsurface. The depth at which the signal amplitude attenuate to 37% (1/e) of its initial value is called the electromagnetic skin depth (δ) and is defined as:

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}} = 503\left(\sqrt{\frac{\rho}{f}}\right)(m)$$

where

δ = skin depth

μ = magnetic permeability

σ = conductivity=1/resistivity

ω = angular frequency=2πf

ρ=resistivity=1/conductivity

In frequency domain, the ratio between the two measured components (E and H) is called electrical impedance (Z) and is defined as $|Z|=|E/H|$. The impedance is a complex number and is used to calculate an apparent resistivity as follows:

$$\rho_a = \frac{1}{\mu\omega}|Z|^2\,(ohm.m)$$

The apparent resistivity is also a function of the frequency. The apparent resistivity can be considered as a volumetric weighted average of the resistivity and thickness of the rocks being sampled. Consequently, it is a smoothly varying function of the frequency. It can be shown theoretically that on a log-log plot of the apparent resistivity vs. frequency the curve cannot exceed a slope of +/- 45 degrees for a layered earth model.

For a homogenous half-space or a one-dimensional (1D) earth model the phase is related to the apparent resistivity through the Hilbert transform. This association does not exist for the 2D and the 3D earth models.

Interpretation of the MT data is performed using the maps of apparent resistivity and the maps of true resistivity of the subsurface. Inversion algorithms in 1D, 2D, or 3D are used to invert the apparent resistivity and phase data in to the maps of the resistivity of the subsurface.

In 1D earth assumption, off-diagonal elements of the impedance tensor are equal and of opposite signs and the diagonal elements are zero. The 1D inversion of the MT data produces a resistivity-depth profile for each site. The results represent a first order approximation of the resistivity variations of the subsurface using a layered-earth model. If there are lateral variations in the resistivity of the subsurface along one direction only (perpendicular to the strike) then a 2D inversion and interpretation is required. In this case, for a data rotated to the strike direction, off-diagonal elements of the impedance tensor are of opposite signs but not equal and the diagonal elements are zero. A cross-section of the true resistivity variations perpendicular to the assumed strike direction is created in the 2D inversion and is used in interpretation. For more complex geological structures a 3D inversion is required to adequately describe the resistivity variation of the subsurface. In this case, none of the elements in the impedance tensor are equal or zero.

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